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Beyond the Semiclassical Description of Black Hole Evaporation

R. Parentani[1]

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In the semiclassical treatment, i.e. in a classical black hole geometry, Hawking quanta emerge from trans-Planckian configurations because of scale invariance. There is indeed no scale to stop the blueshift effect encountered in the backward propagation toward the event horizon. On the contrary, when taking into account the gravitational interactions neglected in the semiclassical treatment, a new UV scale could be dynamically engendered and could stop the focusing. To show that this is the case, we use the large-N limit, where N is the number of matter fields. In this limit, the semiclassical treatment is the leading contribution. Nonlinear gravitational effects appear in the next orders and in the first of these, the effects are governed by the two-point correlation function of the energy–momentum tensor evaluated in the vacuum. In this case they can also be obtained by considering light propagation in a stochastic ensemble of metrics whose mean fluctuating properties are determined by this two-point function.

KEY WORDS: black hole evaporation; quantum gravity; large-N limit

1. OUTLINE: THE LARGE-*N* LIMIT

This paper prolongs our contribution to the former Peyresq meeting (Parentani, 2001a). To ease the reading, we have kept it self-contained. Therefore certain parts have not been modified. However, in many places the text have been changed, and many discussions and equations have been added.

Our aim is to compute the quantum gravitational corrections to Hawking radiation (Hawking, 1975). Since we do not have a theory of quantum gravity, the first task is to choose an approximative treatment which allows to compute radiative corrections to some physical quantities. In this paper, we have chosen a statistical treatment based on a large-N limit, where N is the number of copies of the matter field. The reason for this choice are the following.

First, the semiclassical treatment *is* the leading contribution in the large-N limit. This is most simply understood in a path integral approach (Hartle and

[1] Laboratoire de Mathématiques et Physique Théorique, CNRS UMR 6083, Université de Tours, 37200 Tours, France; e-mail: parenta@celfi.phys.univ-tours.fr.

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Horowitz, 1981): By performing a saddle point approximation in the evaluation of the one-loop effective action, one verifies that the location of the saddle is determined by the semiclassical Einstein equations. Hence, the semiclassical treatment is a mean field approximation (Hartree) in which the N copies of the radiation field propagate in a self-consistent classical geometry: a solution of Einstein equations driven by N times the mean value of the energy–momentum tensor, the one-point function $\langle T_{\mu\nu} \rangle$.

This result can also be understood from a diagrammatic point of view in the following way. One first verifies that the expectation value of any observable can be expanded as a *double* series in G and N in which the power of N is always smaller or equal to that of G. One then verifies that the semiclassical value of this observable corresponds to the resummed series containing all terms governed only by the one-point function $\langle T_{\mu\nu} \rangle$. Moreover, all these graphs are one-particle reducible and their weight is a positive power of GN. This analysis furnishes an alternative proof that the semiclassical treatment is the leading contribution and that it corresponds to a mean field treatment (see the Appendix for more details).

This diagrammatic analysis naturally leads to inquire about the next series. Upon having first summed up the leading series in powers of GN, one encounters a next series containing positive powers of $G^2 N$. This second series is governed by the connected two-point function $\langle T_{\mu\nu}(x)\ T_{\alpha\beta}(x') \rangle_{\mathrm{C}}$, the 'specific heat' of the radiation field. The second reason of having chosen the large N limit is that non-perturbative effects can be obtained by resumming this second series (Parentani, 2001b). In fact the central point of this work is to further analyze this procedure.

The third reason which has led us to choose a treatment based on a statistical basis follows from the so-called trans-Planckian problem. Indeed, in the presence of the unbounded growth of frequencies encountered near the event horizon, perturbative treatments of radiative corrections seem inappropriate (Parentani, 1999).

Let us briefly explain what is the nature of the trans-Planckian problem and why radiative corrections induced by quantum gravity might give some important effects when applied to Hawking radiation (or to quantum effects induced by the presence of an event horizon). When studying the *origin* of Hawking quanta one faces a difficulty which is specific to horizon physics: The configurations giving rise to Hawking quanta are characterized by ultrahigh frequencies when measured by infalling observers near the horizon (Jacobson, 1991; Massar and Parentani, 1996; 't Hooft, 1990, 1996; Unruh, 1981). Indeed, in the semiclassical treatment, the use of free fields propagating in a given background leads to *unbounded* frequencies as a direct consequence of the structure of the outgoing null geodesics near the horizon. Therefore, *any* UV scale which would signal the breakdown of the semiclassical treatment will be *inevitably* reached.

This reasoning is nicely illustrated by considering sound propagation in an acoustic geometry which possesses a horizon: One then finds that the propagation is dramatically modified with respect to the standard propagation (governed by the d'Alembertian) when the UV scale (the interatomic distance) is reached. In particular, for all nonlinear dispersion relation the focusing stops when the new scale is reached. The second point which should be emphasized is that this dramatic modification of the near horizon propagation leaves no imprint on the asymptotic properties of Hawking radiation when the inverse interatomic distance is well separated (Brout *et al.*, 1995a; Unruh, 1995) from κ, the surface gravity of the hole.

Our aim is to show that similar results are obtained when computing nonperturbatively the gravitational effects driven by the connected two-point function $\langle T_{\mu\nu} T_{\alpha\beta}\rangle_{\mathrm{C}}$. We shall find that the trans-Planckian correlations which existed in the semiclassical treatment are washed out when the $r - 2M$ reaches $\bar{\sigma}$, the *new* length scale which plays the role of the interatomic distance. In the simple model we shall use, we obtain $\bar{\sigma} \propto \kappa$ in Planck units. Moreover this washing out mechanism leaves the asymptotic properties of Hawking radiation unaffected: the thermal flux receives corrections which scale like $(\kappa\bar{\sigma})^2$ and which are therefore negligible for large black holes.

2. INTRODUCTION

In his original derivation (Hawking, 1975), Hawking considered the propagation of the radiation in a *fixed* background metric, that of a collapsing star. This means that the metric is once for all determined by the energy of the collapsing star. It is therefore unaffected by the quantum processes under examination. In this approximation, the radiation field satisfies a linear equation (in the absence of matter interactions). One then finds that the infalling and outgoing field configurations are completely uncorrelated near the black hole horizon. This is explicitized by the fact that the connected part of the two-point correlation function of T_{uu} and T_{vv}, the energy–momentum tensors of outgoing and infalling[2] configurations, i.e.,

$$\langle T_{vv}(x) T_{uu}(x')\rangle_C = \langle T_{vv}(x) T_{uu}(x')\rangle - \langle T_{vv}(x)\rangle\langle T_{uu}(x')\rangle, \tag{1}$$

identically vanishes in the vacuum. Nevertheless, Hawking radiation is pair creation. This is perfectly consistent with Eq. (1) since the pairs are composed of two outgoing quanta, one of each side of the even horizon. The external ones form the asymptotic flux whereas their partners fall toward the singularity at $r = 0$; see Massar and Parentani (1996) for a detailed description of the space–time properties of these pairs. Upon tracing over the inner configurations one gets an outgoing incoherent flux described by a thermal density matrix.[3]

[2] v and u are radial advanced and retarded null coordinates. In the Schwarzschild geometry, they are given by $v = t + r^*$ and $u = t - r^*$, where $r^* = r + 2M \ln(r/2M - 1)$ is the tortoise coordinate (Misner *et al.*, 1973).

[3] Moreover, one can explicitize the EPR correlations amongst configurations living on each side of the horizon by computing $\langle T_{uu} T_{uu}\rangle_{\mathrm{C}}$. From Carlitz and Willey (1978) and Masser and Parentani (1996), one gets $\langle T_{uu}(u) T_{uu}(u'_{\mathrm{L}})\rangle_C \propto |u - u'_{\mathrm{L}} + i\pi/\kappa|^4$, where $e^{\kappa u_L} = \kappa U_{\mathrm{K}} > 0$, with U_{K} being the

From this fixed background description, one may go one step further by performing a mean field approximation, i.e. by including the the metric change determined by Einstein's equations driven by the expectation value $\langle T_{\mu\nu}\rangle$. One then finds that this expectation value is *regular*. This is important as it guarantees that the black hole will adiabatically evaporate while keeping the regularity of the near horizon geometry. This regularity in turn implies that the infalling and outgoing configurations will stay *uncorrelated*. Then, as in a fixed metric, $\langle T_{vv}\langle T_{uu}\rangle_C$ vanishes in the semiclassical treatment.

This adiabatic evolution would provide a reliable starting point for including perturbatively radiative corrections if another feature of black hole physics was not present. Namely, the field configurations giving rise to Hawking quanta possess arbitrary high (trans-Planckian) frequencies near the horizon. When measured by infalling observers at r, the frequency of an outgoing quantum of asymptotic energy λ grows as

$$\omega \propto \frac{\lambda}{1 - 2M/r}. \tag{2}$$

This implies that a wave packet centered along the null outgoing geodesic u had a frequency $\omega \propto \lambda\, e^{\kappa u}$ when it emerged from the collapsing star. Unlike processes characterized by a typical energy scale, the relation $\omega \propto \lambda e^{\kappa u}$ shows that black hole evaporation rests, in this scenario, on arbitrary high frequencies. This analysis of wave packets is confirmed by the study of the nondiagonal matrix elements of $T_{\mu\nu}$ which encode the fluctuations of the flux around its mean value. As shown in Massar and Parentani (1996), contrary to the expectation value (the diagonal part) which is regular and of the order of M^{-4}, these matrix elements are generically *singular* on the horizon, i.e., their Fourier content is characterized by frequencies ω which grow according to Eq. (2).

As emphasized by 't Hooft (1990, 1996), this implies that the gravitational interactions between the configurations giving rise to Hawking quanta and infalling quanta cannot be neglected, thereby questioning the vanishing of Eq. (1).[4] In questioning the validity of the semiclassical description, two issues should be distinguished (see Section 3.7 in Brout *et al.*, 1995b). First, there is the question of the low frequency $O(\kappa)$ changes which can be measured asymptotically, and secondly, that of the high frequency modifications of the near horizon physics. Since all thermodynamical reasonings indicate that the asymptotic properties (namely

usual Kruskal retarded time and $\kappa = 1/4M$ being the surface gravity. (It fixes Hawking temperature $T_{\mathrm{H}} = \kappa/2\pi$. We work in Planck units: $c = \hbar = M_{\mathrm{Planck}} = 1$.) The smooth maximum of this two-point function for opposite points, i.e., $u = u'_{\mathrm{L}}$ or $U_{\mathrm{K}} = -U'_{\mathrm{K}}$, is a direct consequence of the fact that the pairs are composed of two outgoing quanta whose asymptotic temperature is $(2\pi/\kappa)^{-1}$.

[4] In Section 9 of the review by 't Hooft (1990, 1996), one reads "Any decomposition of Hilbert space in terms of mutually noninteracting field quanta will be hopelessly inadequate in this (near horizon) region."

thermality governed by κ and stationarity) should be preserved, the problem is to conciliate their stability with the radical change of the near horizon physics which is needed to tame the trans-Planckian problem. This is not an easy problem: Indeed, a naive perturbative analysis (Parentani, 1999) of near horizon interactions leads to back-reaction effects which grow like ω in Eq. (2). This threatens the stationarity of the flux and therefore questions the choice of the treatment which is adequate to go beyond the semiclassical approximation.

As a first step toward a full quantum gravitational treatment, inspired by 't Hooft (1990, 1996), Massar and Parentani (1996), Brout *et al.* (1995b), Kiem *et al.* (1995), Casher *et al.* (1997), Hu and Shiokawa (1998), Martin and Verdaguer (2000), and Barrobis *et al.* (2000), we propose to apply a nonperturbative treatment based on the connected two-point function $\langle T_{\mu\nu}(x) T_{\alpha\beta}(x')\rangle_C$. As mentioned in the Outline, this treatment emerges in a large-N limit. In physical terms, in this limit, infalling vacuum configurations act as an environment for the outgoing quanta and their mutual gravitational interactions express themselves in terms of a stochastic ensemble of metric fluctuations. The specification of vacuum at early times determines the statistical properties of this ensemble and this, combined with the nontrivial properties of the black hole metric, introduces a new scale $\bar{\sigma}$ (in terms of κ) and provides a frame which breaks the 2D local (Jacobson, 1991) Lorentz invariance.[5] Then, the main effect of these interactions is to dissipate the trans-Planckian modes near the horizon but without affecting the asymptotic properties of Hawking radiation.

The unsolved question concerns the range of validity of the treatment based on the two-point function of $T_{\mu\nu}$. This is a complicated question whose final answer requires a better understanding of quantum gravity. Let us nevertheless make some remarks. First, this question closely follows that concerning the validity of the semiclassical treatment which is *equally* complicated.[6] Second, our analysis indicates that the semiclassical treatment fails before our treatment. "Before" should be understood radially, given the blueshift effect encountered during the backward propagation of configurations specified on $\mathcal{J}^+$, see Eq. (2). What emerges is a kind of Russian doll structure in which gravity progressively dominates the physics. Far

[5] In the vicinity of a black hole horizon, free propagation is governed by a 2D Lorentz (and scale) invariance in the u, v plane: near the horizon the 4D d'Alembertian reduces to $\partial_u \partial_v \phi = 0$ see Eq. (7), and the Green function is a sum of functions of $u - u'$ and $v - v'$ separately. When including radiative corrections, the dressed propagator no longer possesses this property. The reasons for this breakdown of Lorentz invariance are similar to those which give rise to the fact that the self-energy of an electron immersed in a thermal bath of photons is not Lorentz invariant: In both cases, in the low energy regime the integrands governing loop corrections are not Lorentz invariant because of the non-trivial metric in the black hole case and the thermal bath in the other.

[6] The validity of the semiclassical treatment has been often questioned in rather general terms. However, a significant answer requires to find physical quantities (i.e., matrix elements of observables) which are incorrectly evaluated in this treatment *and* to propose improved expressions for the same quantities in order to see the discrepancy. We shall provide an explicit exemple in Section 6.

away from the hole ($r - 2M \gg 2M$) one has outgoing thermal (on shell) radiation. In a first intermediate regime ($\bar{\sigma} \ll r - 2M \ll 2M$) the propagation of quanta is still governed by the d'Alembertian but observers at fixed r and free falling ones perceive them differently. It is in this regime that Hawking radiation gets established, (see Eqs. (76)–(83) in Massar and Parentani, 1996. This description based on modes stops to be valid when reaching Jacobson's time-like boundary (Jacobson, 1993), at $r - 2M \simeq \bar{\sigma}$, when outgoing modes get progressively entangled to the infalling configurations, thereby loosing their "mode" quality. The principal aim of this paper is to analyze this transitory regime. Deeper in $r - 2M$, one has some unknown regime governed by Planckian physics. This physics presumably also occurs around us but stays well hidden inside its Planckian husk in the absence of a good microscope.

2.1. The Relation With Sonic Black Hole Physics

For the interested reader, we now further discuss the relationships between our approach and the physics of sonic black holes. The starting point of this new approach to black hole physics is the analogy with condensed matter physics pointed out by Unruh (1981) (and revisited by Jacobson, 1991, 1993). Unruh noticed that sound propagation in a moving fluid obeys a d'Alembert equation which defines an acoustic metric. Therefore, when the acoustic metric corresponds to that of a collapsing star, thermally distributed phonons should be emitted. However, contrary to photons the dispersion relation of phonons is not linear for frequencies (measured in the rest frame of the fluid) higher than a critical ω_c. Since the frequencies $\omega > \omega_c$ which were solicited in Hawking's derivation are no longer available, the stationarity of the flux is directly threatened. However this does not occur: when $\omega_c \gg \kappa$ the asymptotic properties of Hawking phonons are unaffected (Brout *et al.*, 1995a; Jacobson, 1996; Unruh, 1995) in spite of the fact that the near horizon propagation of the phonon field drastically differs from that of photons when the blueshifted frequency reaches ω_c which acts like $\bar{\sigma}^{-1}$. The appealing feature of these models is to provide both, a simple explanation (in terms of adiabaticity which essentially follows from scale separation $\omega_c \gg \kappa$) for the robustness of the asymptotic properties of the flux, and a simple physical reason (a modified dispersion relation) which eradicates the ultrahigh frequencies. (It should be pointed out that a similar trans-Planckian problem arises in inflationary models when studying the origin of the spectrum of primordial energy density fluctuations (Jacobson, 1999; Martin and Brandenberger, 2001; Niemeyer, 2001). In that case as well, scale separation and regularity of the metric are sufficient conditions to guarantee that the properties of the spectrum are unmodified (Niemeyer and Parentani, 2001).)

Besides the robustness of the IR properties, the main outcome of these considerations is that a *new universality* has emerged: for *all* dispersion relations but the linear one, the blueshift effect stops. Therefore, the never ending blueshifting effect obtained by using the linear (scaleless and nondispersive) relation $\Omega = p$ now appears as an isolate and unstable behavior. Thus, instead of asking if Hawking radiation is robust against modifying the dispersion relation, we are let to question: is $\Omega = p$ robust ? or it is simply an artifact of free field theory?

These considerations suggest that quantum gravity should engender a new UV scale when evaluating radiative corrections in a black hole geometry. This new scale would then break the scale invariance of free field propagation and prevent the appearance of trans-Planckian frequencies (Brout *et al.*, 1995b; Jacobson, 1996; Masser and Parentani, 1996). To verify this conjecture, one must determine the physical effects induced by the *nonlinearities* engendered by gravitational interactions. When this is done, one can indulge in the luxury of making contact with dumb holes physics (Velicky, xxxx) by analyzing if/why the effects of nonlinearities can be reproduced by an effective linear equation (i.e., a nontrivial dispersion relation) governing outgoing propagation.

3. THE MODEL

For simplicity, we shall consider only s-waves propagating in spherically symmetric space–times. For definiteness, the background metric is taken to be that resulting from the collapse of a null shell of mass M_0 which propagates along the null ray $v = 0$. Inside the shell, for $v < 0$, the geometry is Minkowski and described by

$$ds^2 = -(1 - \frac{2M(v)}{r})\, dv^2 + 2\, dv\, dr + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{3}$$

with $M = 0$. Outside the shell, the metric is Schwarzschild and given by Eq. (3) with $M(v) = M_0$. As we shall see, this choice of the collapsing metric will have no influence in what follows since we shall focus on the vacuum interactions occurring near the horizon, outside the collapsing matter.

To identify the degrees of freedom involved in these interactions, we first analyze the global properties of radial modes, when working in the geometric optic approximation, i.e., when working with $\partial_u \partial_v \phi = 0$. (In the exact d'Alembertian, see Eq. (7), there is a potential around $r = 3M$ which induces partial reflection, a phenomenon irrelevant for our purposes.) The ingoing massless waves fall into two classes according to their support on $\mathcal{J}^-$, the light-like past infinity. The waves in the first class have support only for $v < 0$, inside the shell, and will be noted ϕ_-. They propagate inward in the flat geometry till $r = 0$ where they bounce off and become outgoing configurations (see Fig. 1.) The relationship between the value of u of the geodesic which originates from v on $\mathcal{J}^-$ is (Barrabès, 2000): $V(u) = -4M(1 + e^{-ku})$. The first class is thus divided in two subsectors: For $v < -4M$, the reflected waves cross the infalling shell with $r > 2M$ and reach the asymptotic region whereas those for $0 > v > -4M$ cross the shell with $r < 2M$

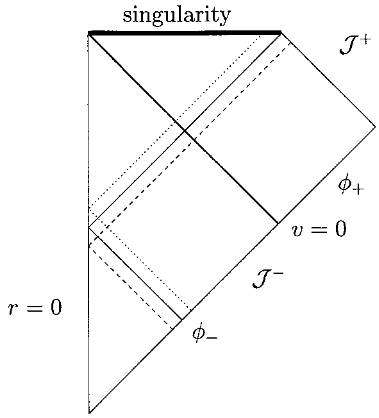

Fig. 1. Penrose diagram of the background geometry. The light-like infalling shell propagates along $v = 0$. The other continuous line emerging from $\mathcal{J}^-$ is $v = v_{\rm H}$, the radial light ray which forms the event horizon after having bounced on $r = 0$. The dashed line represents a characteristic of the configurations ϕ_- which are responsible for Hawking quanta. The dotted line represents a partner's characteristic. For quanta reaching $\mathcal{J}^+$ at late u, both of these characteristics are extremely close to $v_{\rm H}$, (see Eq. (5)). The configurations ϕ_+ have support for $v > 0$ and are always infalling. In this paper, we shall study the interactions between ϕ_- and ϕ_+ which occur outside the star in the near horizon empty region, and when the state of ϕ_+ is vacuum.

and propagate in the trapped region till the singularity. The separating light ray $v_{\rm H} = -4M$ becomes the future horizon $u = \infty$ after bouncing off at $r = 0$. The configurations which form the second class live outside the shell, have support only for $v > 0$ and are noted ϕ_+. They propagate in the static Schwarzschild geometry, are always infalling and cross the horizon toward the singularity.

In Hawking's derivation of black hole radiation, the field operator also obeys the d'Alembert equation. Hence the above classical (on-shell) properties also apply: The configurations for $v < v_{\rm H}$ give rise to the asymptotic quanta and those for $v_H < v < 0$, to their partners (Massar and Parentani, 1996) whereas ϕ_+ plays no role in the asymptotic radiation. For further details concerning the properties of Hawking radiation, we refer to the review by Brout *et al.* (1995b). One also finds that the correlations between the asymptotic quanta and their partners follow from the fact that, on $\mathcal{J}^-$ and in the vacuum, the rescaled field $\phi = \sqrt{4\pi r^2}\chi$ (where χ is the 4D s-wave) satisfies

$$\langle \phi(v)\phi(v')\rangle = \int_0^\infty \frac{d\omega}{4\pi\omega}\, e^{-i\omega(v-v')} = -\frac{1}{4\pi}\ \ln(v - v' - i\epsilon) + \text{constant}. \quad (4)$$

Since this equation is valid for all v, v' one might think that there also exist correlations between ϕ_- and ϕ_+. However, for late Hawking quanta, they effectively vanish since these quanta and their partners emerge from configurations which are localized extremely close to v_H. This follows from the asymptotic ($\kappa u \gg 1$) behavior of the relation $V(u)$

$$V(u) - v_H \propto e^{-\kappa u}. \quad (5)$$

As shown in Hawking (1975), this exponential is responsible for the thermal radiation at temperature $\kappa/2\pi$. It also shows that the quanta emerge from trans-Planckian frequencies on $\mathcal{J}^-$ since $\omega\, dV = \lambda\, du$, where $\omega = i\partial_v$ on $\mathcal{J}^-$ and $\lambda = i\partial_u$ on $\mathcal{J}^+$.

In brief, in the absence of gravitational interactions, ϕ_- and ϕ_+ are effectively two independent fields. By independent we mean that by sending quanta described by wave packets built with ϕ_+ only, there is *no* induced emission of Hawking quanta. Indeed, in order to get induced emission (Wald, 1976) at time u, one should send ϕ_- quanta localized close to v_H as indicated in Eq. (5) and correspondingly characterized by frequencies $\omega \propto \lambda\, e^{\kappa u} \gg \kappa$.

Let us now analyze more closely how these properties translate in the Fock space of ϕ. When evaluated in the background metric (3), the action of ϕ is

$$S_g^\phi = -\int dv dr \left[\partial_v \phi \partial_r \phi + \left(1 - \frac{2M}{r}\right) \frac{(\partial_r \phi)^2}{2} \right] \tag{6}$$

with $M(v) = 0$ for $v < 0$ and $M(v) = M_0$ for $v > 0$. Being interested in the near horizon physics, we have dropped the potential term of s-waves, $(2M_0/r^3)\phi^2$, since it does not affect the near horizon propagation. This can be seen by using the double null coordinate system $u = v - 2r^*$, v. Using them, the 4D d'Alembertian reads

$$\left[\partial_u \partial_v - \left(1 - \frac{2M_0}{r}\right) \left(\frac{l(l+1)}{r^2} + \frac{2M_0}{r^3} \right) \right] \phi_l = 0, \tag{7}$$

where ϕ_l is the rescaled mode of angular momentum l. Thus, as emphasized in Kiem *et al.* (1995), the propagation of waves (at fixed angular momentum and even for an arbitrary mass) effectively obeys a 2D conformal invariance in the near horizon geometry.[7] This is confirmed by the fact that, classically, the trace of 2D part of $T_{\mu\nu}$ vanishes *off-shell*. Thus, in our model for s-waves, $T_{\mu\nu}$ has only two q-number components, $T_{vv} = (\partial_v \phi)^2$ and $T_{uu} = (\partial_u \phi)^2$.

When considering ϕ in second quantization, the 2D conformal invariance implies that the Fock space is composed of tensorial products of infalling states (on which ϕ_+ acts) and outgoing states. In a Schroedinger language this means that an initially factorized state (i.e., $|\Psi\rangle = |\Psi_+\rangle \otimes |\Psi_-\rangle$) remains factorized when its evolution is governed by Eq. (6). This absence of entanglement between the two sectors explains the above-mentioned absence of induced emission when adding a few ϕ_+ quanta.

In a Heisenberg language, it tells us that any matrix element of ϕ is a combination of matrix elements of ϕ_- and ϕ_+ evaluated separately. This implies in particular that the connected part of the two-point correlation (1) identically vanishes for all factorized states. This applies to the "Unruh" vacuum, the state describing Hawking radiation. Physically, the vanishing of Eq. (1) means that the *fluctuations* of T_{vv} and T_{uu} around their mean values are completely uncorrelated. This

[7] This invariance leads to the trans-Planckian problem: the steady production rate of outgoing quanta arises from an integral over in-frequencies ω whose measure is that of a 2D massless field. Explicitly one obtains that the thermal distribution is multiplied by $d\omega/\omega = \kappa\, du$ since $\omega \propto e^{\kappa u}$ (see Eq. (2)). For more details see Parentani (1999) or Eq. (2.54) in Brout *et al.* (1995b).

is just another way to say that there cannot be induced emission of ϕ_- by ϕ_+. It should also be clear that this absence of quantum correlations is precisely what is contested by t'Hooft (see Footnote 3).

Finally, in spite of this absence of correlation, we mention that the mean value of T_{vv} and T_{uu} are related to each other by energy conservation through the 2D trace anomaly (Davies *et al.* 1976). However, this new component of $T_{\mu\nu}$ does not fluctuate: it is a c-number. Hence it cannot play any role in the gravitational interactions between ϕ_- and ϕ_+.

4. THE GRAVITATIONAL INTERACTIONS BETWEEN ϕ_- AND ϕ_+

The aim of this section is to obtain the dominant part of the action governing the gravitational interactions between ϕ_- and ϕ_+. In the next sections, we shall study the consequences of these interactions with particular emphasis on the correlations they induce.

The generating functional governing our matter-gravity system is

$$Z = \int \mathcal{D}\phi\, \mathcal{D}h\, e^{i[S^{\phi}_{g+h} + S_{h,g}]}. \tag{8}$$

h is the change of the metric with respect to the background g discussed above and $S_{h,g}$ is the Einstein–Hilbert action. S^{ϕ}_{g+h} is the action of ϕ propagating in the fluctuating geometry $g + h$.

When the metric fluctuations are spherically symmetric, h can be characterized by two functions: ψ, and μ. Moreover both are completely determined by the energy–momentum tensor of ϕ. This is like the longitudinal part of the electromagnetic field which is constrained to follow the charge density fluctuations, by Gauss' law. The line element in the fluctuating metric can be written as (Barrabès *et al.*, 2000) [8]

$$ds^2 = e^{\psi}\left[-\left(1 - \frac{2\tilde{M}}{r}\right)dv^2 + 2dv\; dr\right] + r^2 d\Omega_2^2 \tag{9}$$

where $\tilde{M} = M_0 + \mu(v, r)$ for $v > 0$. In this new metric, the matter action is the

[8] This line element differs from that used by Bardeen (1981):

$$ds^2 = e^{\psi}\left[-e^{\psi}\left(1 - \frac{2M_0 + 2\mu_B}{r}\right)dv^2 + 2\,dv\,dr\right] + r^2 d\Omega_2^2.$$

The ψ function is the same whereas, to first order in ψ and μ_B, $\mu = \mu_B - \psi(r - 2M_0)/2$. The usefulness of our choice is that ψ no longer affects the null geodesics. We also recall that Einstein's equations read $\partial_v \mu_B = T_{vv} - T_{uu}$ and $\partial_{r^*}\psi = 4T_{uu}/(r - 2M)$, when expressing $T_{\mu\nu}$ in terms of the null fluxes T_{vv}, T_{uu}.

same as in Eq. (6):

$$S^{\phi}_{g+h} = -\int dv\, dr \left[\partial_v \phi \partial_r \phi + \left(1 - \frac{2\tilde{M}}{r}\right) \frac{(\partial_r \phi)^2}{2} \right]. \tag{10}$$

The new mass function $\tilde{M}$ incorporates the only relevant metric change μ. Indeed, S^{ϕ}_{g+h} is independent of ψ, thereby recovering the 2D conformal invariance mentioned earlier.

Our aim is to work out the first-order corrections due to the gravitational interactions between ϕ_- and ϕ_+. To this end only quadratic terms in h should be kept in $S_{h,g}$. The Gaussian integration over h can be performed (this is equivalent to solve the linearized Einstein's equations). It gives rise to a self-interacting field theory described by

$$Z = \int \mathcal{D}\phi\, e^{i S^{\phi}_{g} + i S_{\text{int}}}. \tag{11}$$

By construction S_{int} is a nonlocal quadratic form[9] of the energy–momentum tensor of ϕ.

To identify the relevant part of S_{int} we first recall that $T_{\mu\nu}$ has only two fluctuating components, thanks to the 2D conformal invariance. Thus, in a perturbative treatment (such as the interacting picture) one obtains two types of interaction terms only. First one has self-interaction terms depending on ϕ_- or ϕ_+ separately. These terms do not destroy the factorizability of the theory and will not be considered in what follows.[10]

Secondly, one has a cross-term coupling ϕ_- to ϕ_+. The essential point is that this term will inevitably break the factorizability of the theory into the $\pm$ sectors. Therefore, the two-point function Eq. (1) will no longer vanish. Let us analyze the cross-term coupling ϕ_- to ϕ_+. Since infalling configurations obey $\partial_r \phi_+ = 0$ even in the presence of gravitational interactions, the cross-term coupling ϕ_- to ϕ_+ is

[9] In the t, r coordinate system, i.e., when $g_{rt} = 0$, S_{int} is given by a linearized version (see Eq. (90) in Massar and Parentani, 1996) of the BCMN Hamiltonian (Berger, 1972).

[10] The validity of this radical simplification (also adopted in Casher *et al.*, 1997; Kiem *et al.*, 1995; 't Hooft, 1990, 1996) requires further analysis. We hope to be able to report on it in the near future. On-shell, the $\phi_+ \phi_+$ contribution to S_{int} vanishes. This can be understood from the fact that the Vaidya metric (2) is an exact solution for any classical infalling massless flux $T_{vv}(v)$. The $\phi_- \phi_-$ contribution to S_{int} is more tricky to handle in the advanced coordinates v, r. The reason is that infalling geodesics are affected by the presence of an outgoing flux T_{uu} (as clearly seen when using the coordinates u, r). This modification translates in v, r into a deformation of the description of outgoing geodesics $u = u(v, r)$ and it is this effect which is responsible for the $\phi_- \phi_-$ contribution to S_{int}. Let us finally notice that a nonperturbative treatment of the self-interactions of ϕ_- has been developed in Kraus and Wilczek (1995) and Massar and Parentani (2000). It leads to small effects $O(\kappa/M)$ and induces no damping of the waves when approaching the horizon.

(see Eq. (10))

$$S_{int} = G \int_0^\infty dr \int_0^\infty dv \, \frac{\mu_+(v)}{r} (\partial_r \phi_-)^2 \tag{12}$$

where G is Newton's constant. We have introduced it in the front of μ to read more easily the order of the interactions between ϕ_- and ϕ_+ in the forthcoming equations. $\mu_+(v)$ is the mass fluctuation driven by ϕ_+. Einstein's equations constrain is to be

$$\mu_+(v) = \int_0^v dv' T_{vv}(v') = \int_0^v dv' \, (\partial_{v'} \phi_+)^2. \tag{13}$$

The reader might be surprised by the fact that we are using on-shell fields $\phi_\pm$ in S_{int}. In principle indeed, only the off-shell field ϕ should be used in the action. However, when calculating perturbatively lowest order corrections in G, this amounts to use Eq. (12) as it stands.

We are now in position to show that the gravitational interactions between ϕ_+ and ϕ_- diverge on the horizon. To this end, let us consider two classical fluxes described respectively by $T_{vv} = \Omega \, \delta(v - v_0)$ and $T_{uu} = \lambda \, \delta(u - u_0)$. Ω and λ are the asymptotic energies measured on $\mathcal{J}^-$ and $\mathcal{J}^+$ respectively, and v_0 and u_0 are such that the two spherical shells meet at r_0 in the near horizon geometry, for $r_0 - 2M \ll 2M$. In this case, using $r_0 - 2M \simeq 2M \, e^{\kappa(v_0 - u_0)}$ one obtains

$$S_{int} \simeq 4G \frac{\Omega \, \lambda}{r_0/2M - 1}. \tag{14}$$

The action S_{int} diverges as $r_0 \to 2M$ like ω did it in (3). The difference with (3) is that S_{int} is a *scalar*. Hence the divergence in Eq. (14) is coordinate (gauge) invariant.

By considering two shells whose energy is Hawking temperature, i.e. $\Omega = \lambda = \kappa$, $S_{int} \simeq 1$ gives us a naive estimate of where the gravitational interactions become strong, i.e., can no longer be ignored. The condition $S_{int} = 1$ is reached for $r_0/2M - 1 = G\kappa^2$ ($= 1/M$ in Planck units). This naive estimate will be recovered in Section 6 when considering radiative corrections in the *vacuum*.

We should perhaps emphasize this last point: even though our approach closely follows that of 't Hooft (1990, 1996) (it can be considered as an s-wave reduction of it) we shall not study the interactions between ϕ_+ and ϕ_- quanta. Rather we shall focus on the residual interactions when the state of ϕ_+ is vacuum. For earlier attempts in this direction, we refer to Kiem *et al.* (1995) and Casher *et al.* (1997). Before analyzing these second quantized effects, it is instructive to solve two preparatory exercises with on-shell fluxes.

In the first we shall show that S_{int} acts only as a shift operator of the asymptotic value of u. In spite of this simplicity, in the second exercise, we show that S_{int} nevertheless engenders an entanglement which prevents the factorizability of the states into $\pm$ sectors. This provides an explicit example of a quantum effect induced by Eq. (12), which is absent in the semiclassical description.

5. NONVACUUM GRAVITATIONAL EFFECTS

Let us consider the following problem: Given ϕ_+^0 on $\mathcal{J}^-$ and ϕ_-^0 on $\mathcal{J}^+$, what is the value of the field amplitude ϕ near the horizon?

Because of the 2D conformal invariance, the scattered amplitude ϕ still decomposes as $\phi_+ + \phi_-$. Then, since $\partial_r \phi_+ = 0$ is exact in our gauge wherein v stays light-like in the presence of gravitational interaction, $\phi_+(v) = \phi_+^0(v)$. Thus the infalling flux of energy is unaffected by the energy carried by ϕ_- and it is given by $T_{vv} = (\partial_v \phi_+^0)^2$. Hence, μ_+ of Eq. (13) acts as a given metric change in the equation of motion of ϕ_-:

$$\left[2\partial_v + \left(1 - \frac{2M_0 + 2G\mu_+(v)}{r}\right)\partial_r\right]\phi_- = 0. \tag{15}$$

Since this equation is linear in ϕ_- and first order in the space–time derivatives, its exact solution is

$$\phi_-(v, r) = \phi_-^0(u_\mu(v, r)), \tag{16}$$

where $u_\mu(v, r)$ is the outgoing null geodesic in the modified metric characterized by $M_0 + \mu_+(v)$. The modified geodesic $u_\mu(v, r)$ also obeys Eq. (15) with the boundary condition that it converges to the unmodified value $u_0(v, r) = v - 2r^*$ for $r \to \infty$. To first order in G, the change $\delta u = u_\mu - u_0$ is determined by a nonhomogeneous equation[11] whose solution is

$$\delta u(v)|_{u_0} = G\int_v^\infty dv' \frac{2\mu_+(v')}{r(v')|_{u_0} - 2M_0}, \tag{17}$$

where $r(v)|_{u_0}$ is obtained by inverting $u_0(v, r) = v - 2r^*$. The important point is that, once more, the integral in Eq. (17) is dominated by the near horizon region where $r(v)|_{u_0} - 2M_0 \ll 2M_0$.

The lesson we got from Eq. (16) is that the eikonal approximation is exact: the scattered value of the field amplitude is given by its asymptotic value evaluated along the modified characteristic $u_\mu(v, r)$. Thus, classically, the gravitational interactions encoded in Eq. (12) only induce a shift of the argument of field and do *not* induce nonlinearities in the field amplitude. (The origin of this miracle is the $2D$ conformal invariance. This is not a new remark; see e.g. Kiem *et al.*, 1995.)

In spite of this absence of nonlinearities in the field amplitude, we shall now prove that the quantum evolution governed by the action $S_g + S_{\rm int}$ dynamically engenders entanglement between the otherwise uncorrelated $\pm$ sectors. To

[11] Using the fact that $2\partial_v + (1 - 2M_0/r)\partial_r$ defines $2\partial_v|_{u_0}$ (by definition of $u_0(v, r) =$ constant), δu obeys $\partial_v|_{u_0}\delta u = (\mu_+/r)\partial_r|_v u_0$, thereby giving Eq. (17).

this end, let us consider the evolution of an initially factorized wave function $|\Psi^{\rm in}\rangle = |\Psi_+^{\rm in}\rangle \otimes |\Psi_-^{\rm in}\rangle$. The infalling part $|\Psi_+^{\rm in}\rangle$ is specified on $\mathcal{J}^-$ for $v > 0$. Moreover, to clearly exhibit the entanglement, we choose $|\Psi_+^{\rm in}\rangle$ to be a superposition of two well-defined and separated wave packets

$$|\Psi_+^{\rm in}\rangle = A|\Psi_+^{{\rm in},a}\rangle + B|\Psi_+^{{\rm in},b}\rangle. \tag{18}$$

By well-defined and separated we mean that the two fluxes associated with each component, $\langle T_{vv}^i\rangle \equiv \langle\Psi_+^{{\rm in},i}|T_{vv}|\Psi_+^{{\rm in},i}\rangle$ with $i = a, b$, are localized and separated from each other. This implies that the overlap between $|\Psi_+^{{\rm in},a}\rangle$ and $|\Psi_+^{{\rm in},b}\rangle$ vanishes.

The outgoing piece of the initial ket, $|\Psi_-^{\rm in}\rangle$, is specified on $v = 0^+$, just outside the infalling matter engendering the black hole. We could have specified it on $\mathcal{J}^-$ for $v_H < v < 0$. However, we have chosen $v = 0^+$ outside the star, for having to deal neither with the reflection on $r = 0$ nor with the interactions with the infalling star matter.

Having specified the initial state, we study the quantum dynamics. As in classical terms, the $\partial_r\phi_+ = 0$ now viewed as an Heisenberg equation tells us that the evolution in the $+$ sector is trivial. Therefore, in the interacting picture, the action of the evolution operator $e^{iS_{int}}$ on the total wave function $|\Psi^{\rm in}\rangle$ will give rise to two uncorrelated evolutions for the $-$ sector: one in the a-modified metric characterized by the c-number mass $M = M_0 + \mu_+^a$ with

$$\mu_+^a(v) = \int_0^v dv' \, \langle T_{vv}^a(v')\rangle, \tag{19}$$

and the other one in the b-modified metric. Thus the final value of the wave function (on the union of the event horizon and $\mathcal{J}^+$) is

$$\begin{aligned} |\Psi\rangle \equiv e^{iS_{int}}|\Psi^{\rm in}\rangle &= e^{iS_{int}}\left[\left(A|\Psi_+^{{\rm in},a}\rangle + B|\Psi_+^{{\rm in},b}\rangle\right) \otimes |\Psi_-^{\rm in}\rangle\right] \\ &= A|\Psi_+^{{\rm in},a}\rangle \otimes e^{iS_{int}^a}|\Psi_-^{\rm in}\rangle + B|\Psi_+^{{\rm in},b}\rangle \otimes e^{iS_{int}^b}|\Psi_-^{\rm in}\rangle. \end{aligned} \tag{20}$$

$S_{\rm int}^i$ are the two interaction hamiltonians acting on $|\Psi_-^{\rm in}\rangle$. They are given by Eq. (12) with the corresponding the metric changes μ_+^i, $i = a, b$.

The entanglement induced by S_{int} acts, as usual, as a measurement: Consider for instance the Stern–Gerlach experiment wherein the center of mass motion is determined by the spin projection of the electron which is moving in a magnetic field. The mapping from that situation to the present one is as follows. The two kets representing the spin projections are here played by the two infalling states $|\Psi_+^{{\rm in},\,i}\rangle$. The center of mass wave function is played by the outgoing wave function $|\Psi_-\rangle$ and the interaction Hamiltonian is S_{int} of Eq. (12). The analogy works quite well when the initial outgoing wave function $|\Psi_-^{\rm in}\rangle$ is well peaked. Then, its "image" on $\mathcal{J}^+$ would be *either* a spot at $u_0 + \delta u_a$ with probability $|A|^2$, *or* one at $u_0 + \delta u_b$ with probability $1 - |A|^2$. u_0 is the location of the spot when the gravitational interactions are ignored and the values of the shifts δu_a, and δu_b are given by Eq. (17) fed by the mass changes μ_+^a and μ_+^b respectively.

This quantum result should be compared with what would have been obtained by using the semiclassical treatment, i.e., by using the *mean* mass change

$$\bar{\mu}_+(v) = |A|^2 \mu_+^a(v) + |B|^2 \mu_+^b(v) \tag{21}$$

instead of each mass change separately, (see Eq. (19)). The semiclassical treatment incorrectly predicts a single spot on $\mathcal{J}^+$ located at the "mean" position $u_0 + \delta\bar{u}$ with $\delta\bar{u} = |A|^2 \delta u_a + |B|^2 \delta u_b$.

The validity of the semiclassical treatment rests on the possibility in neglecting the fluctuations in the operator-valued mass function $\hat{\mu} = \int dv\, \hat{T}_{vv}$, which is determined by the fluctuation of the infalling flux $\hat{T}_{vv}$, and which determines in turn the fluctuations of the operator-valued δu through Eq. (17). Thus, the physical importance of the mass fluctuations is governed on one hand by the connected part of two-point function $\langle T_{vv} T_{vv} \rangle_C$ which characterizes the fluctuations of T_{vv}, and on the other by the amplification of the effects through the denominator in Eq. (17). Even the relative importance of $\langle T_{vv} T_{vv} \rangle_C$ with respect to the mean square flux $\langle T_{vv} \rangle^2$ is small in the two evolutions, the exact one and the semiclassical differ, i.e., the semiclassical treatment leads to incorrect predictions, when the fluctuations are sufficiently amplified by the denominator in Eq. (17). Instead in the absence of amplification, the mean theory is always correct, unless $\mu_a(v) - \mu_b(v))/M \simeq 1$, but in this case the notion of a classical background completely fails.

In brief, the crucial point is the following: Unlike what one encounters in usual circumstances, i.e., without an event horizon, tiny fluctuations of T_{vv} might give rise to large shifts in u because of the amplification due to the growth of the gravitational interactions when the configurations meet near an event horizon.

5.1. Relationship With Former Treatments

Before considering second quantized effects, it is also interesting to relate Eq. (11) to the former treatments of black hole evaporation discussed in the literature: Hawking's approach (Hawking, 1975) and the semiclassical treatment.

Hawking's approach characterized by a fixed geometry is recovered by putting $G\mu_+ = 0$ in Eq. (11). Then Z factorizes as $Z_+ \otimes Z_-$ (when ignoring the trace anomaly) and ϕ_- is a free outgoing field propagating in the background geometry g. Thus ϕ_+ drops out from *all* matrix elements built with the operator ϕ_-. It should be emphasized that the trans-Planckian problem (*e.g.*, the fact that the *in–out* Green function is characterized by trans-Planckian frequencies when one of the operator approaches the horizon; (Massar and parentani, 1996; Barrabès *et al.* 2000)) encountered in Hawking's approach directly follows from this factorizability. Indeed, it is the *absence* of gravitational coupling between the

$+$ and $-$ sectors which permits the unbounded growth of frequencies when probing, near the horizon, configurations specified on $\mathcal{J}^+$.

The semiclassical treatment (Bardeen, 1981; Massar, 1995; Parentani and Piran, 1994) can be obtained from the path integral formalism (8) by first integrating over ϕ at fixed h and then searching for the classical extremum of h. In this approach, by construction, the fluctuations of h and $T_{\mu\nu}$ are neglected. Thus the near horizon propagation of ϕ is governed by a *self-consistent* metric governed by mean $\langle \mu_+(v) \rangle$. This mean evolution characterizes by the shrinking of the horizon area according to

$$\frac{d\langle \mu_+(v)\rangle}{dv} = \langle T_{vv} \rangle|_{r=r_{\text{horizon}}=2M}. \tag{22}$$

When working in the vacuum equation (4), the (properly subtracted; (Brout *et al.*, 1995b)) expectation of T_{vv} is

$$\langle T_{vv}(v) \rangle|_{r=2M(v)} = -\frac{\pi}{12}\left(\frac{\kappa(v)}{2\pi}\right)^2, \tag{23}$$

where $\kappa(v) = (4M(v))^{-1}$ with $M(v) = M_0 + G\langle \mu_+(v) \rangle$. This flux has the opposite value of a 2D thermal flux. The only change with respect to the fixed background approach of Hawking is the replacement of M_0 by $M_0 + G\langle \mu_+ \rangle$. Thus the propagation of outgoing configurations is hardly affected by the evaporation as long as it is slow, i.e., as long as $M(v) \gg M_{\text{Planck}}$.

Therefore, in the semiclassical scenario, the trans-Planckian problem stays as in Hawking's approach: the coupling between ϕ_- and the mean change $\langle \mu_+ \rangle$ is incapable to provide a taming mechanism since it does not open new interacting channels. To solve this problem clearly requires to take into account the *fluctuating* character of the interactions between ϕ_- and ϕ_+, i.e., the possibility of entangling their wave functions, as in the quantum mechanical exercise presented above.

6. MODIFIED GREEN FUNCTION

Our aim is to show how the two-point Green function of ϕ_-

$$G(x_1, x_2) = \frac{\int \mathcal{D}\phi \phi_-(x_1)\phi_-(x_2)\, e^{iS_g + iS_{int}}}{Z}, \tag{24}$$

where Z is given in Eq. (11), is affected by the gravitational interactions encoded in S_{int} when the infalling configurations are in their vacuum state.[12]

[12] Being in the search of a new scale induced by vacuum effects, we should reconsider the simplification of having kept only the $+, -$ interaction term in S_{int}, *c.f.*, the discussions after Eq. (11). The neglect of the other terms might indeed fallaciously engender the new scale; more on this delicate problem when considering the value of the cutoff, after Eq. (31).

To evaluate Eq. (24) beyond the semiclassical treatment, one should adopt some rules to cope with the UV divergences. The scheme we propose consists in considering N copies of ϕ. The calculation of Eq. (24) can then be achieved in two different approaches. The first consists integrating first over the $N-1 \simeq N$ spectator copies not appearing in the numerator in so as to determine the influence functional (IF) (Feynman and Hibbs, 1965) governing the effective dynamics of ϕ. The other consists in developing $e^{iS_{\rm int}}$ in both integrands of Eq. (24) in powers of $S_{\rm int}$ so as to engender the (connected) Feynman diagrams governing the radiative corrections.

The usefulness of the IF approach is that nonlinear effects are naturally taken into account through the effective action for ϕ. The same nonlinear effects can of course be reached from the Feynman diagrams approach at the cost of summing infinite subsets of graphs. It is in the *identification* of these infinite subsets that the large N limit finds its real justification. For a schematic description of these diagrammatic aspects, we refer to the Appendix. In what follows, we shall pursue with the IF approach.

When computing the lowest order corrections to the self-energy in Eq. (24), we can use Eq. (4), the "free" propagator of ϕ_+. This approximation concerning degrees of freedom *not* directly involved in the matrix elements (i.e., which factorized out in the absence of interactions) is a common procedure both in quantum field theory where it gives the vacuum contribution, (see Chapter 9 in Feynman and Hibbs, 1965), and in statistical mechanics (*e.g.*, the *polaron*, Chapter 11). In our case, in this approximation, the IF gives rise to a nonlocal action which is a sum of terms containing $(\partial_r\phi_-)^2$ and kernels given by the Wick contractions of T_{vv} evaluated with Eq. (4).[13] To order G^2, one obtains

$$S_{\rm IF} = i\langle S_{\rm int} S_{\rm int}\rangle_+ = iG^2N \int d^2x \int d^2x'\,(rr')^{-1}(\partial_r\phi_-)^2\,\langle\mu_+(v)\mu_+(v')\rangle(\partial_{r'}\phi_-)^2, \tag{25}$$

where $\langle\quad\rangle_+$ means that the expectation value applies to ϕ_+ only. Using Eq. (4), the connected two-point function is

$$\langle T_{vv}(v)\; T_{vv}(v')\rangle_C = \frac{1}{16\pi^2}\frac{1}{(v-v'-i\epsilon)^4}\,. \tag{26}$$

Then, Eq. (13) gives

$$\langle\mu_+(v)\mu_+(v')\rangle = \frac{1}{96\pi^2}\frac{1}{(v-v'-i\epsilon)^2}$$

[13] In general, when one does not make this approximation, the Wick contractions of ϕ_+ will give rise to a series in powers of G which starts with Eq. (26) and whose higher order terms depend on ϕ_-. Thus Eq. (26) would have become operator-valued (Kiem *et al.*, 1995) in ϕ_-, thereby obtaining a situation analogous to that of transition amplitudes when enlarging the phase space so as to take into account recoil effects Parentani, 1995.

$$= \frac{1}{96\pi^2} \int_0^\infty d\omega\, \omega \exp[-i\omega(v - v')]. \tag{27}$$

This equation gives the mean metric fluctuations driven by ϕ_+ in the unperturbed ($G = 0$) vacuum state; (see Martin and Verdaguer (2000) for a analysis of the 4D two-point function of induced metric fluctuations in Minkowski vacuum. Notice that $\langle \mu_+(v)\mu_+(v')\rangle$ is not real. This follows from the quantum vacuum which contains only positive frequencies when hit by ϕ (see Eq. (4)). Notice that Eq. (26) gives the "unsubtracted" value of the connected two-point function. As shown in Tomboulis (1977), the counterterms which lead to the renormalized one-point function (23) also provide divergent contributions to eq. (26) which tame its singular behavior as $v \to v'$.

Keeping only Eq. (25) in the IF (or by summing the corresponding infinite set of Feynman graphs; see the Appendix) is equivalent to work with a stochastic (i.e., a classically given) Gaussian ensemble of metric fluctuations. By "equivalent" we mean that all matrix elements of ϕ_-, such as Eq. (24), can be computed from this stochastic theory. Thus as far as the propagation of ϕ_- is concerned and to lowest order in G, the functional integration over ϕ_+ in Eq. (24) effectively engenders a stochastic ensemble of metric fluctuations governed by Eq. (27).

Hence all the techniques developed in Barrabès *et al.* (2000) apply. In what follows we shall present schematically the main results and we refer to this work for details. The key point is the following. Because of the Gaussianity of the ensemble, one can obtain *nonlinear* corrections to Eq. (24) from the fluctuating characteristics of Eq. (15), i.e., the outgoing null geodesics $u_\mu(v, r)$, the non-trivial solutions of $ds^2 = 0$ in the fluctuating metric (9). In this we recovered that there is no nonlinearities in the field amplitude: as in Section 5 the scattering only occurs through the characteristics.

To determine the effects engendering these metric fluctuations, it is instructive to analyze the backward in time propagation of asymptotic plane waves representing Hawking quanta. The reason for this is that, in a fixed background, the Fourier transform of the *in–out* Green function performed on $\mathcal{J}^+$ obeys

$$\int du\, e^{i\lambda u} G_{\text{in–out}}(u, v = \infty; v, r) \propto \phi^*_\lambda(v, r) \propto e^{i\lambda u(v,r)}. \tag{28}$$

In the absence of metric fluctuations, $u(v, r) = u_0(v, r) = v - 2r^*$. Hence near the horizon the plane wave $e^{-i\lambda u}$ behaves as

$$e^{-i\lambda u_0(v,r)} \simeq \theta(r - 2M_0)\, e^{-i\lambda v}(r - 2M_0)^{i\kappa\lambda}. \tag{29}$$

It vanishes for $r < 2M_0$ and possesses an infinite number of oscillations as $r \to 2M_0$ with increasing momentum $p_r = -i\,\partial_r\, e^{-i\lambda u_0(v,r)} = 4M_0\lambda/(r - 2M_0)$. This is the trans-Planckian problem.

In a Gaussian ensemble of metric fluctuations the averaged waves are given by

$$\langle e^{-i\lambda u_\mu(v,r)}\rangle = e^{-i\lambda u_0(v,r)}\, e^{-\frac{\lambda^2}{2}\langle \delta u(v)\delta u(v)\rangle}. \tag{30}$$

Using Eq. (17), the fact that $r_0(v)|_{u_0} - 2M_0 \simeq 2M_0\, e^{\kappa(v-u_0)}$, and Eq. (27), one obtains

$$\begin{aligned}\langle \delta u(v)|_{u_0}\delta u(v)|_{u_0}\rangle &= \frac{G^2N}{(r/2M_0-1)^2}\int_0^\Lambda \frac{d\omega}{3}\left[\frac{\kappa^2\omega}{\kappa^2+\omega^2} + i\,\frac{\kappa\omega^2}{\kappa^2+\omega^2}\right] \\ &= \bar{\sigma}_\Lambda^2\,\frac{1}{(r/2M_0-1)^2} + i(\bar{\sigma}_\Lambda^{\rm disp})^2\,\frac{1}{(r/2M_0-1)^2}.\end{aligned} \tag{31}$$

The spread $\bar{\sigma}_\Lambda$ governs the damping of the backward propagated wave and is equal to $G\kappa\sqrt{N\,\ln(\Lambda/\kappa)/3}$ when the UV cutoff Λ satisfies $\Lambda \gg \kappa$. We have introduced Λ to define the two integrals over ω. Notice that it is a Lorentz scalar, since it is the energy of an s-wave in its rest frame. The imaginary contribution in Eq. (31) defines $\bar{\sigma}_\Lambda^{\rm disp}$ and controls the *dispersion* of the wave, i.e., the λ dependent modifications of the locus of constructive interferences. When $\Lambda \gg \kappa$, one obtains $\bar{\sigma}_\Lambda^{\rm disp} \simeq G\sqrt{N\kappa\Lambda}$.

The first result of Eq. (31) is that $\bar{\sigma}_\Lambda$ is not proportional to Λ even though $\langle \mu_+^2\rangle \simeq \Lambda^2$. Notice also that $\bar{\sigma}_\Lambda$ hardly depends on the value[14] of Λ since $\bar{\sigma}_{\Lambda=4M} = \sqrt{2}\bar{\sigma}_{\Lambda=1}$. This is because high frequencies ($\omega \gg \kappa$) are damped by the integration over v' in Eq. (17). The frequencies $\omega \simeq \kappa$ dominate the real contribution in Eq. (31).

The second result of Eq. (31) is that $\langle \delta u \delta u\rangle$ diverges as $r \to 2M_0$. Thus the correlations between asymptotic quanta and early configurations, which existed in a given background as shown in Eq. (29), are washed out by the metric fluctuations once $r - 2M_0 \simeq \bar{\sigma}_\Lambda \simeq 1/M_0$. The reason of this loss of coherence is that the state of ϕ_- becomes correlated to that of ϕ_+ ('t Hooft, 1990; Kiem *et al.*, 1995). Physically, this loss of coherence implies that induced emission (Wald, 1976) no longer exists when the threshold energy $1/\bar{\sigma}$ is reached.[15] Phenomenologically this loss can be viewed as a dissipation of outgoing waves, and, as in condensed

[14] In the simplified treatment we are using, the value of Λ must be chosen from the outset. Instead, in an improved treatment where all terms are kept in $S_{\rm int}$, we believe that the values of $\bar{\sigma}, \bar{\sigma}^{\rm disp}$ shall be unambiguously determined. In that case, when using the properly subtracted (Tomboulis, 1977) two-point function $\langle T_{\mu\nu}T_{\alpha\beta}\rangle$, only $\omega \simeq \kappa$ will contribute to the $\bar{\sigma}$'s. Indeed in the UV domain all expressions become Minkowskian in character and hence cannot contribute to Lorentz breaking effects such as those engendered by $\bar{\sigma}$. This result can be implemented in our truncated model by putting $\Lambda \simeq \kappa$ in Eq. (31), thereby obtaining $\bar{\sigma} \simeq \bar{\sigma}^{\rm disp} \simeq G\kappa$.

[15] An interesting and unsolved question raised by this loss is whether *new* correlations are induced by the gravitational interactions as the same time as the old ones are washed out by them. This is presently under study.

matter (Hu, 1998; Velicky, 1999), it can be described by a non-trivial dispersion relation, (see Eqs. (5.9) and (5.11) in Barrabès *et al.*, 2002).

We should further explain the physical relevance of these results. To this end, one must identify the matrix elements of ϕ_- which are highly sensitive to the metric fluctuations (and governed by the ensemble averaged waves (30)) and those which are not. The simplest example of an operator which is sensitive is the *in–out* Green function with one operator at v, r and the other on $\mathcal{J}^+$. Indeed since the 'second' point lives on $\mathcal{J}^+$ where the *out* vacuum is defined, the phase of the out-wave function evaluated at fixed u is not affected by metric fluctuations. On the contrary that of the wave function evaluated near the horizon at v, r is sensitive to the metric fluctuations encountered from $\mathcal{J}^+$ to where it lives.[16] It is this (*unusual*; see below) discrepancy in the modification of the phase at each point which explains why the ensemble averaged one-point waves Eq. (30) govern this two-point Green function.

Instead *usual* expectation values, such as, for instance, the *in–in* Green function with two points evaluated at fixed u on $\mathcal{J}^+$ (or two nearby points close to the horizon), are *not* severely affected by the metric fluctuations because they are not governed by the ensemble-averaged waves (30). The reason is that the ensemble average is performed *after* having computed the operator for each member of the ensemble. (This is not a choice: the stochastic ensemble is merely a tool to reproduce quantum mechanical expectation values. This quantum origin fixes the rules of ensemble averaging without ambiguity.). In our case, this implies that the shift Eq. (17) affects *coherently* the phase at each point (see Section IV.A in Barrabès *et al.*, 2000). This guarantees that the shift drops out in the coincidence point limit. This cancellation in turn guarantees that the asymptotic properties are (almost) unaffected since the Green function possesses the usual Hadamard singularity (Freedenhagan and Haag, 1990). By "almost" we mean that the corrections scale like $(\kappa\bar{\sigma})^2$ and thus are of order $1/M^4$. It should be pointed out that it is the dynamically induced scale $\bar{\sigma}_\Lambda$ and not the UV cutoff Λ which governs these corrections.

We would like to further emphasize the fact that the metric fluctuations strongly affect the correlations between configurations specified on $\mathcal{J}^+$ and near

[16] One might wonder if the effects we are describing are not induced by the choice of working at fixed u or at fixed v, r. To waive his qualms, we recall that Green functions have no physical meaning per se, rather they are elements which appear in integrals describing transition amplitudes (for a discussion of this point in a quantum gravitational context See Section 2 in Parentani, 1997). It is through this channel that one can verify that u is a physically meaningful coordinate on $\mathcal{J}^+$ since $du|_r = dt$, where dt is the proper time of a particle detector on $\mathcal{J}^+$. Thus when additional quantum mechanical systems are coupled to the radiation field, the matrix elements governing transition amplitudes will have, in their integrand, phase factors behaving like $e^{-i\lambda u}$ in *any* coordinate system. Similarly, upon questioning what an infalling observer might see when crossing the horizon, v, r are meaningful since $dr|_v \propto -d\tau$ where $d\tau$ is his proper time.

the horizon without modifying the short distance behavior of the Green function.[17] The radical difference of the impact of vacuum gravitational interactions follows from the fact that asymptotic observers inevitably use *out*-states to probe the physics. Therefore, the overlaps they consider will be automatically of the *in–out* type since the Heisenberg state of the field is specified (prepared) before the collapse. It is this two-states formalism giving rise to nondiagonal matrix elements (Massar and Parentani, 1996) (exactly like in an *S*-matrix formulation; (Hooft, 1990, 1996) which is at the origin of the difference: the metric fluctuations cannot coherently affect configurations specified in the 'ket' on $\mathcal{J}^-$ and in the 'bra' on $\mathcal{J}^+$; hence the coherence is lost. (Notice that when computing induced emission probabilities, one automatically considers overlaps between *out*-states specified on $\mathcal{J}^+$ and some early state specified on $\mathcal{J}^-$. This explains why induced emission probabilities are washed out.) On the contrary, infalling observers have access only to the near horizon behavior of the Green function in terms of *in* configurations. Hence coherence is maintained for them.

7. CONCLUSIONS

We have studied the effects induced by the gravitational interactions governed by Eq. (12). Even though we worked out only the lowest order in G ($\bar{\sigma}_\Lambda \propto G$) we believe that our main result is robust: We see no reason for higher order terms to *suppress* the entanglement of ϕ_- with ϕ_+ so as to give $\bar{\sigma}_\Lambda = 0$, thereby *recovering* trans-Planckian correlations. Indeed higher order modifications to Eqs. (26) and (27) should be of the type $(G\omega^2)^n$ and therefore will not affect the low frequency behavior of Eq. (27), thereby leaving the effective spread $\bar{\sigma}_\Lambda$ essentially untouched. Moreover, when considering the effects of higher angular momentum modes, as indicated by Casher *et al.* (1997), $\bar{\sigma}$ should be *larger* than our estimate based on s-modes because the effects of higher angular momenta should add incoherently.

In brief, given that gravitational interactions grow without bound near the horizon, we claim that the entanglement of ϕ_- with ϕ_+ is unavoidable and universal. (By universal we mean that a similar entanglement would also be found when considering the coupling of ϕ to other quantum fields such as massive ones.) The entanglement will then prevent the unbounded growth of frequencies encountered in the free field theory and will be accompanied by the reorganization of the description of vacuum. That is, when $r \to 2M$, the usual states of the free field theory

[17] This clearly illustrates that the physics seen by infalling observers completely differs from that reconstructed from observers at large distance from the hole. This is similar to what was advocated in Kiem *et al.* (1995). However, the fact that the *in–out* Green function obeys Eq. (30) indicates that the near horizon physics is unaccessible and therefore lost to remote observers. Thus it seems that these two descriptions could not obey the "complementarity principle" (Davies *et al.*, 1976). By "complementary to each other," it was meant that the two descriptions are both complete, like the position and momentum representations of the same vector state in quantum mechanics.

which give rise to the notion of on-shell asymptotic particles provide bad approximations of the true eigenstates (still characterized by the Killing energy λ since the situation is stationary) of the interacting theory. It is the growing discrepancy which leads to the ‘dissipation’ of the amplitude in Eq. (30).

We also claim that the radiative corrections encoding dissipation are finite because only low frequency $\omega \simeq \kappa$ infalling configurations contribute to them. Indeed, in the UV regime for both infalling and outgoing configurations, the expressions coincide with those evaluated in the tangent plane and are Minkowskian in character. Hence the high frequency regime cannot contribute to the effects which break Lorentz invariance. (This still needs to be confirmed by an explicit calculation and is presently under examination.)

We would like to conclude this work by several remarks on related aspects of quantum gravity and black hole physics.

First, we point out the similarity between the above effects induced by the metric fluctuations representing gravitational interactions in the vacuum and those attributed (Garay, 1998) to “foam.” By foam we mean quantum gravitational configurations which radically affect the smoothness of space–time at short distance. (They might arise from gravitational instantons (Hawking, 1978) or even stringy effects (Amati, 1989).) In all cases, the replacement of free field propagation in a fixed background by the appropriate interacting model might lead to very similar (universal?) deviations when analyzing the departure from the free field description that all models possess at large distances. Therefore, these first deviations might be described by some effective mesoscopic theory of space–time properties which would essentially signal the existence of a minimal resolution length (Kempf and Mangano, 1997), the equivalent of our $\bar{\sigma}$, in the otherwise local field theory.

Second, we conjecture that $\bar{\sigma}$ (properly computed so as to include the contribution of higher angular momentum modes) should *also* be the length scale which enters in the entanglement description of the black hole entropy (Jacobson, 1994). We recall that when using free field in a given space time, the entanglement diverges owing to the unbounded character of the reservoir of high energy modes. To get a finite entropy density per unit area, some cutoff should be introduced. We propose that the cutoff defining the black hole entropy should be the dynamically induced lengthscale $\bar{\sigma}(N)$, i.e., the length scale at which correlations between configurations on $\mathcal{J}^+$ and the near horizon region get lost when N quantum fields contribute to the entropy and therefore to the near horizon gravitational interactions.

APPENDIX: THE LARGE-*N* LIMIT

We briefly mention several interesting features of the large N limit which illuminate the problems we addressed.

The semiclassical description of quantum processes occurring in a curved background is based on the following equations:

$$G_{\mu\nu} = 8\pi G \,\langle\Psi|T_{\mu\nu}|\Psi\rangle \tag{32}$$

$$\Box_g \hat{\phi} = 0 \tag{33}$$

In Eq. (33) the field operator propagates in the classical geometry $g = g^{\Psi}_{\mu\nu}$ which is a solution of Eq. (32) driven by the expectation value $\langle\Psi|T_{\mu\nu}|\Psi\rangle$ evaluated in the state $|\Psi\rangle$ using Eq. (33). In this Sense, Eqs. (32) and (33) are self-consistent (Hartree) approximations.

It is quite reasonable that this approximation correctly predicts the time evolution of *certain* quantities in *certain* circumstances, *e.g.*, the *rate of mass loss* of a *large* black hole. However, the criteria which characterize the validity range of the predictions obtained from Eqs. (32), and (33) are not known. An obstacle in finding these criteria is the identification of the "small parameter(s)" which control the deviations between the exact evolution and that predicted by the semiclassical equations.

A rather formal answer to these questions is provided by considering a large-N limit, where N is the number of copies of the ϕ field. The simplest way to understand why the semiclassical equations govern the large-N limit is by considering the path integral approach of matrix elements. By duplicating N times ϕ in Eq. (8) and first integrating over them with a fixed metric h, the one-loop effective action for h contains an overall prefactor N when Newton's constant G is expressed as $G = \xi/N$. Therefore, in a large N limit, the path integral over h can be evaluated by a saddle point approximation. The stationary phase condition then gives rise to Eq. (32) and the spread around the saddle point scales like $N^{-1/2}$. In this vision, the validity of the semiclassical equations purely relies on a statistical argument, as in a thermodynamic limit. The weakness of this very general argument is the absence of a hierarchy of the length scales governing the processes under examination, unlike what is found when a Born–Oppenheimer treatment is applied to quantum gravity (Massar and Parentani, 1998).

More interestingly the large-N limit also makes predictions *beyond* the semiclassical equations. For instance, in the limit $N \to \infty$ with GN fixed, the short distance behavior of the graviton propagator is modified (see Martin and Verdaguer, 2000; Tamboulis, 1977). In Minkowski vacuum, these modifications are of course Lorentz invariant. However, in a thermal bath or a curved background, the correction terms will no longer possess the Lorentz invariant form. Hence they can induce the effects we are seeking: a dynamically induced scale which breaks the (local) Lorentz invariance that the uninteracting theory possessed. Moreover, only low energies (i.e., energies comparable to the temperature) can contribute to this new scale because in the UV limit all expressions tend to their Minkowski vacuum, and hence Lorentz-invariant, form. Hence there should not be any additional UV divergences in the expressions giving rise to the new scale, as it is the case for the corrections to the self-energy of an electron immersed in a thermal bath.

To further strengthen the relations with what we did in Section 6, it is instructive to see how the semiclassical treatment emerges from Eq. (24) viewed as generating perturbatively the connected Feynman diagrams when expanding $e^{iS_{int}}$ in powers of S_{int}. In this description, one finds that the Green function is a double sum of powers of N and G which possess the following properties:

- The power of N is equal or inferior to that of G.
- The semiclassical treatment corresponds to the leading series: the set of graphs weighted by $(GN)^n$. All graphs are one-particle reducible and they are governed by the one-point function $\langle T_{\mu\nu} \rangle$. Upon summing this series, one identically recovers the Green function evaluated in the "mean" geometry $g^{\Psi}_{\mu\nu}$, the solution of Eq. (32). In this series all graphs are one-particle reducible and governed by the one-point function $\langle T_{\mu\nu} \rangle$.
- Having summed up the leading series in $(GN)^n$, our treatment corresponds to the *next* series: the set of graphs weighted by $(G^2 N)^n$. All graphs are two-particle reducible and they are governed by the connected two-point function $\langle T_{\mu\nu} T_{\alpha\beta} \rangle_C$. Upon summing this new series, one obtains the Green function evaluated in the stochastic ensemble governed by Eq. (27). This second series should also be related to the use of the above-mentioned large-N modified graviton propagator in the place of the unperturbed one. (We are presently trying to prove this point.)

In brief, N is a parameter which organizes the double sum of graphs into a sum of nonperturbative series whose mth series containing all powers of $(G^m N)$ is governed by the mth correlation function of $T_{\mu\nu}$ when the former series have been first summed up.

The physical question raised by these results is the following: given the dimensionality of $G = l^2_{\text{Planck}}$, can one infer that high orders in m (the relative power of G with respect to that of N) become relevant only for high (Planckian) energies? We conjecture that this is the case: the sorting out of graphs in terms of m is effectively an expansion in the energy of the processes involved in the matrix element under consideration. This is what seems to emerge from our analysis. As indicated by Eqs. (30) and (31), the semiclassical description of the correlations breaks down when $r - 2M \simeq \bar{\sigma}$, i.e., when the energy of a mode $p_r = \lambda/(r/2M - 1)$ reaches the new scale $1/\bar{\sigma}$.[18] We thus find, as in Massar and Parentani (1998), that the validity of the semiclassical equations relies on a *hierarchy* of energy scales. Indeed, for a large black hole, $\bar{\sigma} \ll M$ even when $N = 1$. Thus N is *not necessary* to

[18] Notice that $\bar{\sigma}$ scales as $N^{-1/2}(GN/M)$. This writing expresses that in the large-N limit at fixed GN, i.e., with an N-independent Hawking flux, $\bar{\sigma} \to 0$ like $N^{-1/2}$, thereby verifying that in this limit the scale which signals the breakdown of the semiclassical description indeed vanishes.

justify the validity of the semiclassical description. It is rather a useful parameter which helps sorting out the different contributions in radiative corrections.

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